Comment on "Evidence of Non-Mean-Field-Like Low-Temperature Behavior in the Edwards-Anderson Spin-Glass Model"

A recent interesting paper [1] compares the low temperature phase of the $3D$ Edwards-Anderson (EA) model to its mean-field counterpart, the Sherrington-Kirkpatrick (SK) model. The authors study overlap distributions $P_{\mathcal{J}}(q)$ and conclude that the two models behave differently. Here we notice that a similar analysis using state-of-the-art, larger data sets for the EA model (generated with the Janus computer [2] in [3]) and for the SK model (from [4]) leads to a very clear interpretation of the results of [1], showing that the EA model behaves as predicted by the replica symmetry breaking (RSB) theory.

The authors of [1] study $\Delta(q_0, \kappa)$, the probability of finding a peak in $P_{\mathcal{J}}(q)$ greater than κ for $q < q_0$. In a RSB system, $\lim_{N\to\infty} \Delta(\kappa, q_0) = 1$. Fig. 5 of [1] shows that, at fixed q_0 and at the same temperature T, Δ grows for the SK model, but seems to reach a plateau for the EA model. In the inset of Fig.1 we show that, considering larger systems ($N \leq 32^3$ as opposed to $N \leq 12^3$ of [1]), Δ clearly grows with N also for the EA model. We use the same value of q_0 as in [1] and $T = 0.703$. Even this simple analysis is sufficient, when one looks at state-of-the-art lattice sizes, to show that Δ has the same qualitative behavior in EA and SK.

Still, it is important to remark that comparing data for different models at the same reduced temperature is not a reliable approach when studying non-universal quantities, that have no reason to be similar. Instead, we should select T such that the disorder-averaged $P(q)$ are similar for the two models. We go beyond the analysis in [1] and offer a simple model for the scaling of Δ. According to the RSB theory, in the large-N limit $P_{\mathcal{J}}(q) = \sum_\gamma W_\gamma \delta(q - q_\gamma)$, where the distribution of the weights W_γ is well understood [5]. For large but finite N a very simple (but effective) model [6] postulates that the weight distribution is unchanged, but the delta functions are smoothed to have a finite height $H(N)$. The resulting $P_{\mathcal{J}}(q)$ will have peaks of height $H(N)W_\gamma$. We can estimate the characteristic height $H(N)$ by fitting the disorder-averaged $P(q_{\mathrm{EA}})$ to the form AN^λ (see [3]). Then $\Delta(\kappa, q_0)$ is the probability of finding a peak with weight $W_\gamma > \kappa/H(N)$, which, for small q_0, can be estimated as $\Delta(\kappa, q_0) \sim [\kappa/H(N)]^{-I(q_0)} = (AN^\lambda/\kappa)^{I(q_0)}$, where $I(q_0) = \mathbb{P}(|q| < q_0)$. $\lambda = 1/3$ for the SK model but $\lambda \approx 0.1$ for the EA model [3]; this difference is the reason for the slower growth of Δ in the EA model.

We show Δ at $T = 0.4$ for the SK model (top) and at $T = 0.703$ for the EA model (middle), where the temperatures are such that $P(0)$ are very similar: q_0 ranges from 0.02 to 0.44 for the larger systems. The curves show a clear universal scaling behavior for large N. In the bottom panel we compare Δ for SK and EA using similar effective volumes, i.e., similar values of AN^λ. Both models are in remarkable quantitative agreement.

The method based on peak detection convincingly shows, contrary to the claims of Ref. [1], that finite-dimensional and infinite-range spin glasses have the same behavior: the only ingredients needed are the use of state-of-the-art system sizes and some care in the choice of parameters when comparing non-universal quantities.

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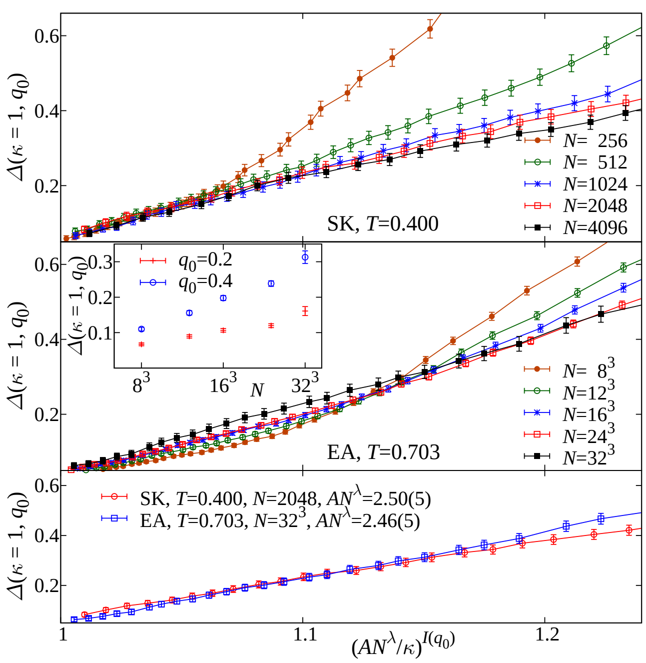

FIG. 1. (color online) $\Delta(\kappa, q_0)$ against $(AN^\lambda/\kappa)^{I(q_0)}$ for SK (*top*) and EA (*middle*). *Inset:* $\Delta(\kappa, q_0)$ for fixed q_0 for the EA model. *Bottom:* comparison of the EA and SK models for similar values of AN^λ.